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Vacuum Stability and the Large-Scale Structure of the Universe

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It is shown that from the assumption that the physical vacuum realizes a state near the maximum of the effective potential it is possible to obtain many of the observed properties of the distribution of galaxies in the universe.

During the recent years several papers have been published (Arnold, 1989; Ellis *et al.*, 1990; Anderson, 1990; Sładkowski, 1991; Mańka and Sładkowski, 1990) discussing vacuum stability. The authors of these papers suggest that we may live in a metastable vacuum. It is even suggested that a state in the maximum of the effective potential may be very stable (Arnold, 1989). If the lifetime of such a vacuum exceeds the age of the universe, this vacuum may be acceptable as a physical one. In this paper the simplest consequences of the assumption that we live in this type of vacuum are discussed. Properties of this vacuum may be discussed in a model-independent way. Specifically, no connection with elementary particle theory is assumed. Near the maximum, the effective potential may be expanded in the power series

$$U(\phi) = U(\phi_0) + \frac{1}{2}\frac{\partial^2 U}{\partial \phi^2}\bigg|_{\phi_0} \Delta\Phi^2 + \cdots \tag{1}$$

In (1), Φ means the effective physical field, which for simplicity is chosen as the scalar field. Using the notation $\Delta\Phi = \Phi - \Phi_0 = \phi$, $\partial^2 U/\partial\Phi^2 = m^2$, and assuming for convenience $U(\Phi) = 0$ (we neglect the cosmological term), we find

$$U(\phi) = \frac{1}{2} m^2 \phi^2 \tag{2}$$

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Now the effective Lagrangian reads

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - U(\phi) \tag{3}$$

which leads to the static Euler–Lagrange equation

$$\Delta\phi = m^2\phi \tag{4}$$

Assuming spherical symmetry, we have

$$\phi'' + \frac{2}{r}\phi' - m^2\phi = 0 \tag{5}$$

In (5) the prime denotes differentiation with respect to the space radius r. Equation (5) has an exact solution of the form

$$\phi = A\frac{e^{-mr}}{r} \tag{6}$$

for $m^2 > 0$, or

$$\phi = A\frac{\sin(mr)}{r} \tag{7}$$

for $m^2 < 0$. The most interesting property of this solution is the surface dependence of the energy (R. Mańka, personal communication, 1991). Using the identity

$$\nabla(\phi\nabla\phi) = (\nabla\phi)^2 + \phi\Delta\phi \tag{8}$$

we find

$$\begin{aligned} E(r) &= \int_V dV\left[\frac{1}{2}(\nabla\phi)^2 + U\right] \\ &= \int_V dV\left[\frac{1}{2}\nabla(\phi\nabla\phi) - \frac{1}{2}\phi\nabla\phi + U\right] \\ &= \int_V dV\left[-\frac{1}{2}\phi\Delta\phi + U\right] + \int_\Sigma d\sigma\,[\phi\nabla\phi] \end{aligned} \tag{9}$$

From the Euler–Lagrange equation (4) it is clear that the volume integral vanishes. This fact is sometimes interpreted as indicating that solutions (6), (7) have zero energy. This is true in the stable vacuum ($m^2 > 0$)—the exponential factor in (6) leads to the vanishing of the surface integral. In the unstable vacuum this is false. The energy density for (7) has the form

$$\mathscr{E} = \frac{1}{2}A^2\left[\frac{m^2}{r^2}\cos(2mr) + \frac{1-\cos(2mr)}{2r^4} - \frac{m\sin(2mr)}{r^3}\right] \tag{10}$$

Integrating (10), we obtain

$$E(r) = \int_V dV\, E = \pi A^2 \left[m \sin(2mr) + \frac{\cos(2mr) - 1}{r} \right] \qquad (11)$$

which really is not zero. For large r, the second term tends to zero, but the first cannot be neglected.

Solution (7) has a clear interpretation: classical excitations of the vacuum state of (3) (with $m^2 < 0$) have the form (7).

A classical (or semiclassical) approach must be especially applicable for the large period r_0. If r_0 has an astronomical value (extremely small m), we can ask about the influence of the field configuration (7) on astronomical bodies. The shape of the energy distribution (11) is simple to find. For example, E is equal to zero for r fulfilling one of the two equations $\sin(mr) = 0$ or $\mathrm{tg}(mr) = mr$, but in fact what is interesting is only the property that the energy density is concentrated in a restricted area.

This energy concentration may be the starting point for the creation of the galaxies.

At this moment, two remarks are needed. First, the fact that the solutions (6), (7) distinguish the point $r = 0$ need not break the large-scale transition symmetry of the universe. In the universe there may exist many excitations of type (7) and galactogenesis may occur at the intersection points of the maximal-energy-concentration spheres.

A second remark is that solution (7) is obtained by neglecting the other terms in (1). This leads to very big regions of large energy density. When the potential is strongly nonlinear these regions may be small.

But the most important question is whether astronomical facts support this approach to the structure of the universe.

The first of these facts is the redshift periodicity (Broadhurst *et al.*, 1990). Some years ago it was found that galaxies distinguish some of the redshifts, which was interpreted as the periodical space distribution of the galaxies. The period of this distribution is about 128 Mpc. Using this value, we can calculate $\partial^2 U/\partial\Phi^2 = m^2$. The value of m is $m = \pi/(128\ \mathrm{Mpc}) = 2.5 \times 10^{-25}\ [1/m]$, or energetically, 10^{-30} eV. In the linear approximation (1) the constant A is arbitrary (in the nonlinear case this is not true) and to obtain its value we need another astronomical fact. Fortunately we have this fact. A large part of the universe, including our galaxy, is moving with respect to the cosmic microwave background. This motion has a clear origin if we assume excitation (7): it is motion in the gravitational field generated by the energy distribution. In the Newtonian approximation, the spherical distribution of the energy (11) leads to the force

$$F = \frac{G\pi A^2 \{m \sin(2mr) + [\cos(2mr) - 1]/r\}}{c^2 r^2} \qquad (12)$$

acting on the material points in the vacuum excitation (7). The change of sign in (12) is due to neglect of the cosmological term in (1). The negative value in (10) and (11) has the same origin. Assuming a positive value of the $U(\Phi_0)$, we must add a term $\int U(\Phi_0)\, dV$ to the entire integral. By comparing (12) with the centrifugal force, we can obtain the value of A:

$$\frac{GE}{c^2r^2} = \frac{v^2}{r} \Rightarrow \tag{13}$$

$$\Rightarrow A^2 = \frac{v^2c^2nr_0}{\pi Gmk} \tag{14}$$

In (13) we chose the value $v = 600$ km/sec (Dressler, 1991) and denoted $k = \sin(nmr_0)$, $r = r_0 n$, $r_0 = 128$ Mpc.

The large value of A may seem strange. In fact, it is only a consequence of bad units, and the physical value of A is rather moderate. Substituting A into (10), we find

$$\mathscr{E} = 1.6 \times 10^{-10} \frac{k'}{kn} \left[\frac{J}{m^3}\right] = 1.8 \times 10^{-30} \frac{k'}{kn} \left[\frac{g}{cm^3}\right] \tag{15}$$

where $k' = \cos(nr_0)$.

It is interesting that the value of the energy density we obtain is very close to the critical density (for k', k and n equal to 1, this density leads to the 30 km/sec · Mpc value of Hubble constant), so the field ϕ may be considered as a good candidate for dark matter, fulfilling all demands of the standard inflationary model. When we notice that solution (7) describes another well-known property of the large-scale structure, that galaxies lie on rather thin surfaces surrounding relatively empty regions (voids), we see that it may be useful to pay some attention to the assumption that our vacuum is a maximum of the effective potential.

Of course, we need not worry about the stability of this maximum. The probability Γ of the tunneling on the unit volume V (Linde, 1983) is proportional to e^{-S}, where S is the Euclidean action described by Lagrangian (3) with solution (7). For the values A and m obtained here Γ is strictly zero. We can obtain the same conclusion using the method developed in Arnold (1989). The critical radius and energy of the bubble may be approximated by r_0 and $10^{16} M_{0\odot}$ respectively. The effective potential is almost constant.

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